Introduction to Aeronautics

ARO 101 Sections 03 & 04

Sep 30, 2015 thru Dec 9, 2015

Instructor: Raymond A. Hudson

Week #5 Lecture Material

Topics For Week #5

- **The Isentropic Equations–** Three equations that relate the static measurements of air temperature, pressure, and density, to their TOTAL counterparts (i.e. with the flow brought isentropically to rest (V=0).
- **Mach Number–** For compressible flow we can derive an equation for measuring Mach number from total and static pressure.
- **True Airspeed–** Derive an equation for compressible true airspeed from the Mach Number equation above.
- **Calibrated Airspeed –** Compressible relationship for calibrated airspeed based on compressible true airspeed equation and assuming standard day, sea level speed of sound and static pressure.
- **Extra Credit Project–** Final chance to sign up!
- **Wind Tunnel Experiment–** Discuss our experiment for next week, look at the test procedures, and talk about test conduct.

Isentropic Equations

Derived From Thermodynamic Relations

ISENTROPIC = "Without heat exchange…no heat transfer"

$$\frac{T_0}{T_1} = 1 + \frac{(\gamma - 1)}{2} M^2$$

- ***Ratio of <u>Total Temperature</u> (of a flow brought to rest isentropically) to <u>Static Temperature</u> of the same flow.***

$$\boxed{\frac{P_0}{P_1} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{\gamma}{(\gamma - 1)}}}$$

- ***Ratio of <u>Total Pressure</u> (of a flow brought to rest isentropically) to <u>Static Pressure</u> of the same flow.***

$$\frac{\rho_0}{\rho_1} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{1}{(\gamma - 1)}}$$

- ***Ratio of <u>Total Density</u> (of a flow brought to rest isentropically) to <u>Static Density</u> of the same flow.***

ARO 101 *Intro-Aero*

Compressible Mach Measurement

And True Airspeed From Isentropic Pressure Ratio

- ***Solving our equation for P_0/P_1 for Mach Number yields:***

$$M^2 = \frac{2}{(\gamma - 1)}\left[\left(\frac{P_0}{P_1}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]$$

- ***Now recall that Mach = Velocity/Speed Of Sound (V_∞/a_{sos})***

$$V_\infty^{\;2} = \frac{2*a_{sos}^{\;2}}{(\gamma - 1)}\left[\left(\frac{P_0}{P_1}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]$$

- ***We can re-arrange via algebra to introduce difference between total and static pressure – TRUE AIRSPEED:***

$$V_\infty^{\;2} = \frac{2*a_{sos}^{\;2}}{(\gamma - 1)}\left[\left(\frac{P_0 - P_1}{P_1} + 1\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]$$

Now Calibrated Airspeed

Eliminate Need to Know Temp for Speed Of Sound

$$V_\infty^{\,2} = \frac{2 * a_{SOS}^{\,2}}{(\gamma - 1)}\left[\left(\frac{P_0 - P_1}{P_1} + 1\right)^{\frac{\gamma}{(\gamma-1)}} - 1\right]$$

- **These two terms give us problems:**
  - **We do not want to have to measure temperature (to compute Speed Of Sound).**
  - **We do not want to have to measure static pressure independent of total pressure (we want airspeed to ONLY be a function of the measurement of the difference between total and static pressure)**

- **Therefore, let us substitute the standard day, sea level values to yield a form of CALIBRATED Airspeed:**

$$V_\infty^{\,2} = \frac{2 * a_{SL}^{\,2}}{(\gamma - 1)}\left[\left(\frac{P_0 - P_1}{P_{SL}} + 1\right)^{\frac{\gamma}{(\gamma-1)}} - 1\right]$$

ARO 101
Intro-Aero

Equivalent Airspeed

Yields the SAME Dynamic Pressure if Airplane Flown at SL STD

- **To reduce the variability of the atmosphere in evaluating aerodynamic performance, we prefer to evaluate said performance with respect to sea-level standard day atmospheric conditions:**

Flight Condition Dynamic Pressure = Sea Level Standard Day Dynamic Pressure

$$\frac{1}{2} * \rho_{\infty} * V_{\infty}^{2} = \frac{1}{2} * \rho_{SL} * V_{E}^{2}$$

- **Solving for V_E and we can always convert true airspeed to equivalent airspeed:**

$$V_E = V_{\infty}\sqrt{\frac{\rho_{\infty}}{\rho_{SL}}}$$

Square Root Of The Atmospheric Density Ratio!