Introduction to Aeronautics

ARO 101 Sections 03 & 04

Sep 30, 2015 thru Dec 9, 2015

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Week #4 Lecture Material

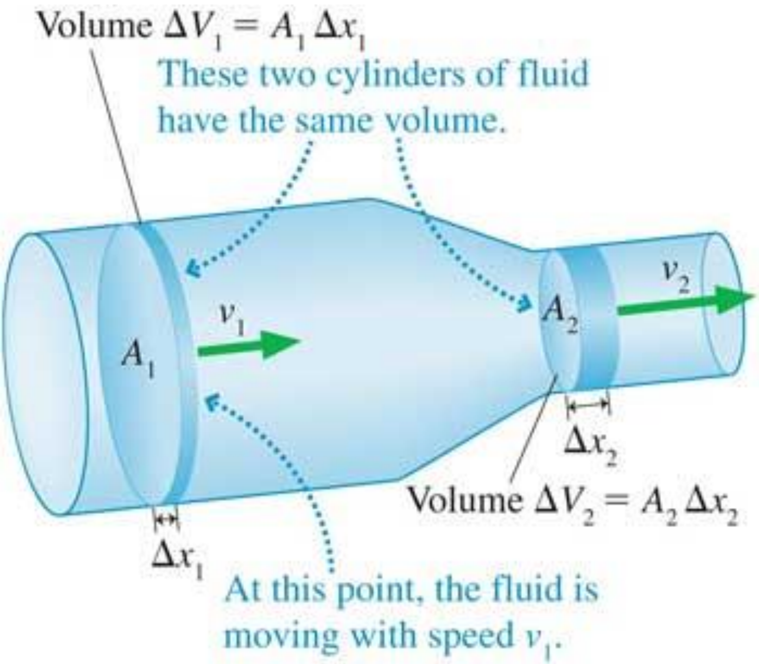
Topics For Week #4

- **<u>The Continuity Equation–</u> When fluid flow is steady (uniform, no surges) through a constrained area (pipe/duct), then one or both types of flow rate remain constant between any two points.**
  - ✓ ***<u>Incompressible (Density Constant):</u>*** *Constant VOLUMETRIC FLOW RATE (**Q**)*
  - ✓ ***<u>Compressible (Density Can Vary):</u>*** *Constant MASS FLOW RATE (**$\dot{m}$**)*
- **<u>Euler's Equation–</u> Answers the question: How does** *pressure vary with velocity* **in a steady, moving fluid flow? Derive analytically.**
- **<u>Bernoulli's Equation–</u> Derived from Euler's Equation but for** *incompressible flow only*! **From this we will *FINALLY* see where dynamic pressure comes from!**
  - ✓ ***<u>Total Pressure, Static Pressure, & Dynamic Pressure Terms</u>***
  - ✓ ***<u>Applying to Wind Tunnel Flow Analysis with Continuity Equation</u>***
- **<u>How Do We Measure Airspeed? –</u> Difference between total and static pressure measurements lead to an airspeed measurement equation. But we must apply correction terms for various reasons! Compressibility effects.**

ARO 101 *Intro-Aero*

Deriving Continuity Equations

Start With Incompressible Water Flowing Out a Nozzle

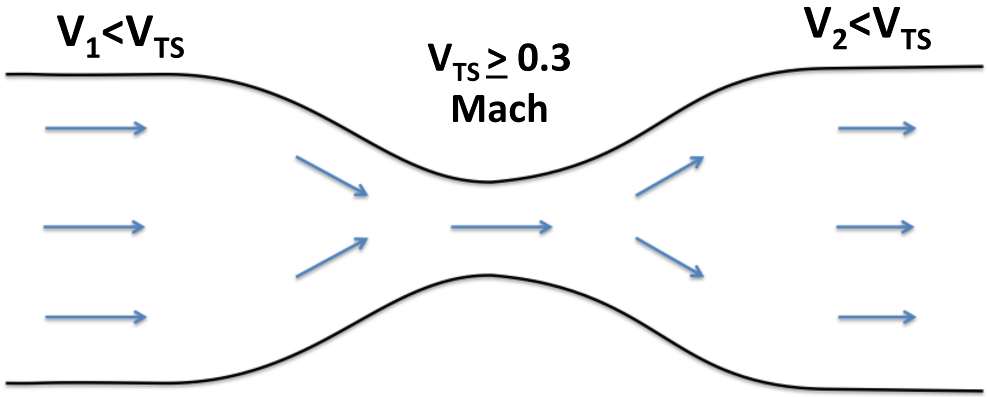


- ***EVERYDAY, INTUITIVE EXAMPLE:***
  1) **The nozzle on your garden hose (when turned on at full, steady, uniform flow) accelerates the water to a higher speed.**
  2) **The smaller the exit orifice of the nozzle, the faster the exit speed.**
- ***DEVELOP THE ENGINEERING EQUATION:***
  1) **Since water is incompressible: volume flow moving into nozzle MUST equal volume flow coming out of nozzle over any period of time (t).**
  2) **Volumetric flow rate = Q = Volume/time**
  3) **Applying to diagram: $Q_1=A_1*\Delta x_1$/time and $Q_2=A_2*\Delta x_2$/time**
  4) **Recognize:** ***Δx/time is VELOCITY***
  5) ***Per observation #1:* $Q_1=Q_2$, *THEREFORE…***

$$\boxed{Q = A_1*V_1 = A_2*V_2}$$

Deriving Continuity Equations

Now Let's Account For Compressibility of Air



- ***As Airspeeds Increase Compressibility Comes Into Play***
  - **While V_1 & V_2 above may not be compressible, the reduced area Test Section may result in airspeeds that result in compressible flow.**
  - **Since density can change, volumetric flow rate (Q) can no longer be considered constant.**
  - ***However, it is still true that the mass flow into a closed channel must equal the mass flow out of the channel.***
  - **Mass Flow Rate ($\dot{m}$) = Density*Area*Velocity = $\rho * A * V_\infty$ (in Slug/Sec or Kg/Sec)**

Compressible Continuity Equation

$$\boxed{\dot{m} = \rho_1 * A_1 * V_1 = \rho_2 * A_2 * V_2}$$

How Does Pressure Vary With Velocity?

Analytical Derivation of Euler's Equation

• **We seek to derive an equation that will describe how fluid pressure varies with respect to fluid velocity. As we did with hydrostatics, we must make some simplifying assumptions:**

1) **The Fluid is LIGHT WEIGHT** – *Air is light compared to liquids like water. The forces that make air move are MUCH larger than the force due to the mass of the air. Therefore, this will allow us to neglect the weight of the air in our analysis.*
2) **The Fluid Flow is INVISCID (Frictionless)** – *Considering the frictional forces between fluid molecules would make this analysis exceedingly complex. Since we are not concerned with drag, as yet, ignoring friction will not have a large impact on our understanding of pressure and velocity.*
3) **The Fluid Flow is LEVEL** – *If the fluid is not moving upwards or downwards, this reinforces assumption #1 so we do not have to consider fluid weight. This also allows us to treat this as a one-dimensional flow problem (left-to-right)*
4) **The Fluid Flow is STEADY** – *As we have discussed already, non-steady flow is a very difficult problem which you will study in later courses.*

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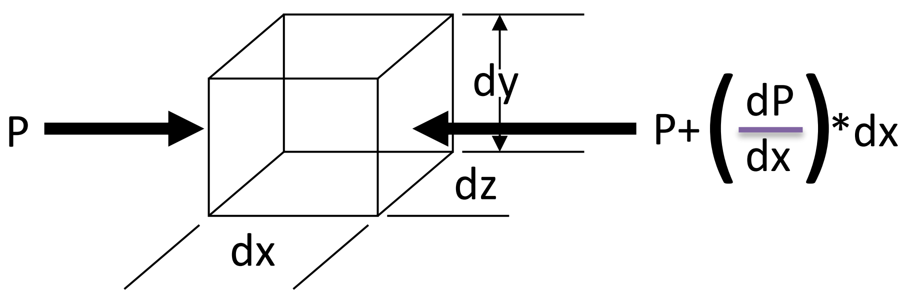
Free Body Diagram Analysis

One Dimensional, Weightless, Inviscid, Steady Flow

- **As we did with the hydrostatic analysis, we begin by drawing our cube of fluid and identifying the forces acting upon it:**



- ✓ **Fluid moving left to right. Ignore forces acting in vertical direction (see assumptions 1 and 3).**
- ✓ **Fluid is moving, so pressures must be unbalanced between left & right faces.**
- ✓ **Pressure must change continuously from one face to the other, thus why we need the (dP/dx) *pressure gradient*.**
- ✓ **Apply Newton's Law: ΣF = ma**

Summing the forces along direction of travel:

$$\Sigma F = P*dy*dz - \left(P+\frac{dP}{dx}*dx\right)*dy*dz$$

Simplify & set resultant force equal to "ma":

$$\Sigma F = -\frac{dP}{dx}*dx*dy*dz = m*a = m*\frac{dV}{dt}$$

$$m = \rho*Vol = \rho*dx*dy*dz$$

$$\frac{dV}{dt} = \frac{dV}{dx}*\frac{dx}{dt} = \frac{dV}{dx}*V$$

Euler's Equation

$$\boxed{dP = -\rho*V*dV}$$

Bernoulli's Equation Derivation

A Special Case (Incompressible Flow) of Euler's Equation

- **IF we assume that the fluid flow is INCOMPRESSIBLE, then Euler's Equation becomes fairly easy to integrate.** *(Note that this now adds a NEW assumption, and therefore a new restriction, with respect to how we may use these results!)*

$$\int_{P_1}^{P_2} dP = -\rho \int_{V_1}^{V_2} V*dV$$

$$(P_2 - P_1) = -\rho*(½*V_2^2 - ½*V_1^2)$$

SEPARATING "1" and "2" CONDITIONS ON OPPOSITE SIDE OF EQUATION YIELDS:

Bernoulli's Equation

$$\boxed{P_1 + ½*\rho*V_1^2 = P_2 + ½*\rho*V_2^2}$$

An Important & Useful FIELD EQUATION

Three Pressure Forms of Bernoulli's

For Steady, Level, Incompressible, Inviscid Flow!

$$\underbrace{P_1 + ½*\rho*V_1^{2}}_{\text{Total Pressure } (P_T) \text{ or } (P_0)} = \text{Static Pressure } (P_s) + \text{Dynamic Pressure } (\bar{q})$$

Static Pressure (P_s): *Pressure measured $\perp$ to fluid flow direction.*

Dynamic Pressure ($\bar{q}$): *AKA "impact pressure." The component of pressure caused by the fluid motion.*

For steady, level, incompressible, and inviscid flow, the TOTAL PRESSURE remains constant along any streamline!

SOME FACTS TO NOTE ABOUT THIS RELATION:

1) ***When there is no flow present ($V_\infty = 0$), then total and static pressure are identical ($P_T = P_S$)***
2) ***For P_T to remain constant along streamline, then as V_∞ increases, P_S must decrease.***
3) ***As V_∞ increases, P_S will decrease with the <u>square of velocity</u>. (That's the magic of flight!)***
4) ***Total pressure (P_T) can be measured by pointing a pressure probe directly into the flow.***

V_∞

This is a Pitot Probe. Attached to outside of many airplanes.

Wind Tunnel Flow Analysis

Using Hydrostatic, Bernoulli's, & Continuity to Perform Useful Analysis!

WIND TUNNEL OFF!

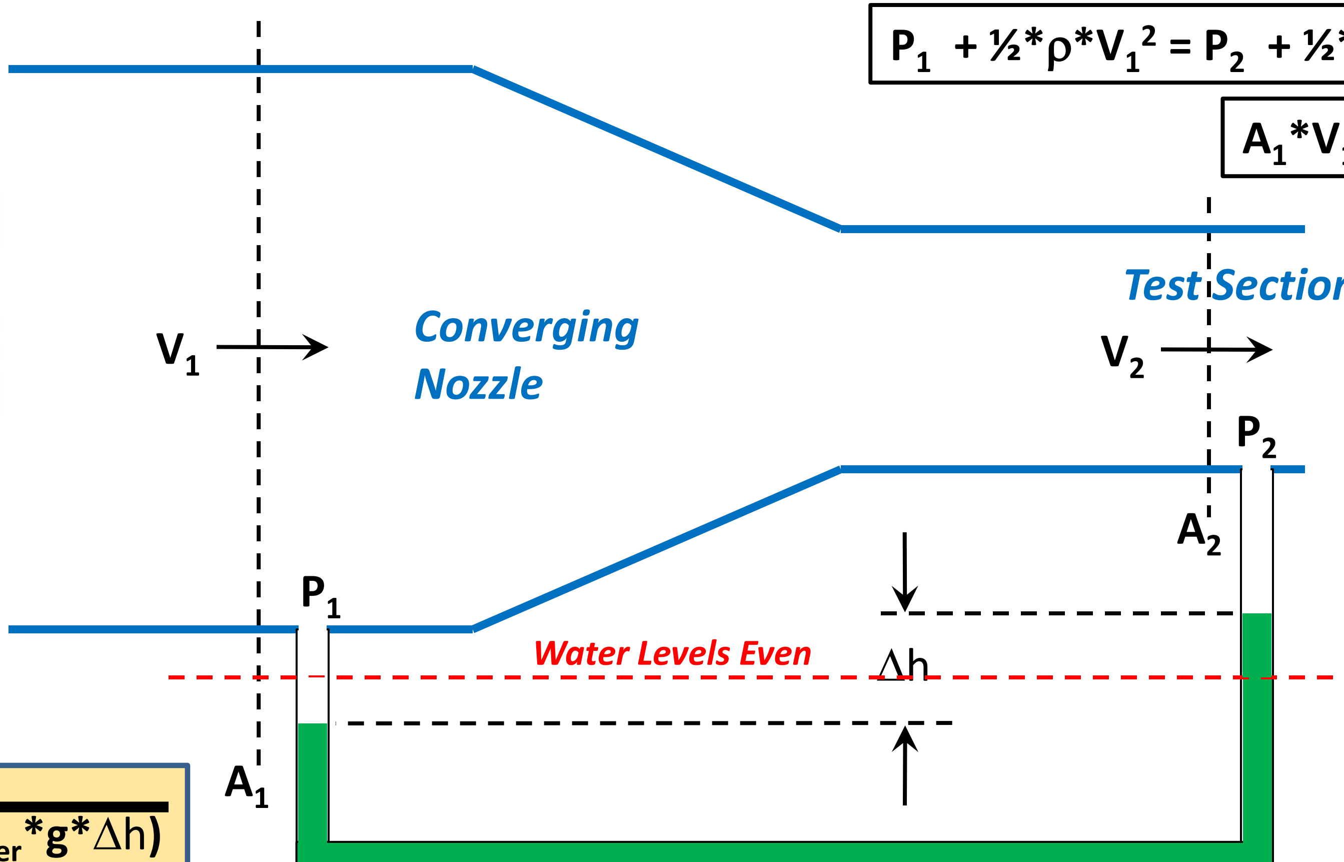
$$P_1 - P_2 = \rho_{water} * g * \Delta h$$

$$P_1 + ½ * \rho * V_1^2 = P_2 + ½ * \rho * V_2^2$$

$$A_1 * V_1 = A_2 * V_2$$

Wind Tunnel

Fan ON

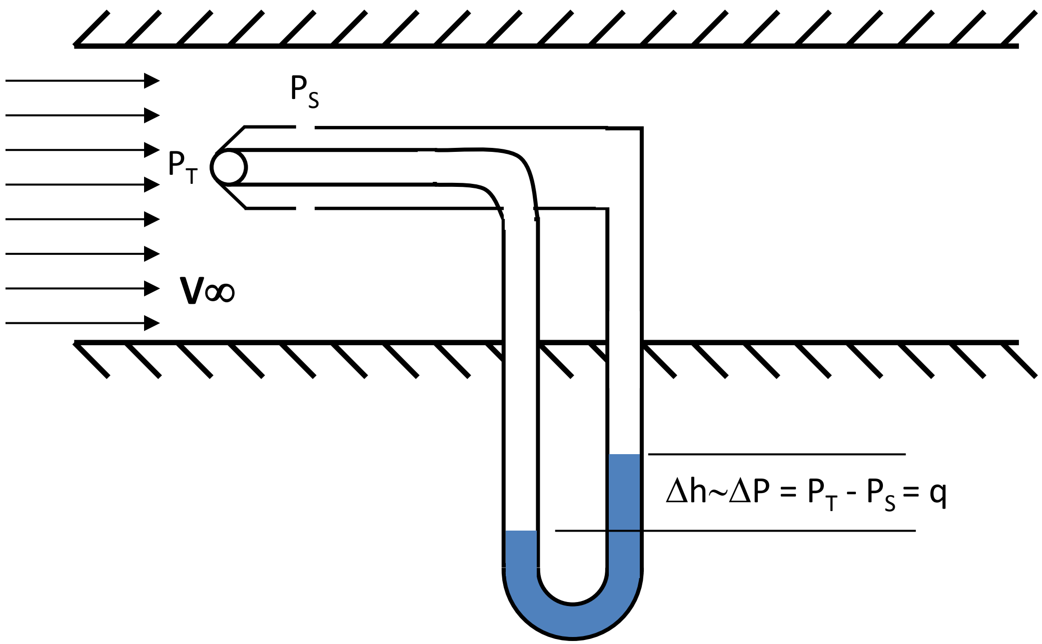


$$V_2 = \sqrt{\frac{2*(\rho_{water}*g*\Delta h)}{\rho*[1-(A_2/A_1)^2]}}$$

Instrument To Measure Airspeed

The Pitot-Static Tube

- **We can configure a single pressure measuring device that measures the difference between freestream TOTAL pressure and freestream STATIC pressure. We call such a device a Pitot-Static Tube.**



$$P_T - P_S = ½*\rho_{AIR}*V_\infty^{\;2} = \rho_{H20}*g_0*\Delta h$$

Solve for V_∞ as a f(Δh)

The Various Forms of Airspeed

"ICeT"

• *Just like we have many different measures of altitude, we also have many different measures of airspeed!*

When expressed in units of knots:

Indicated	**KIAS**	*Knots Indicated Air Speed*
Calibrated	**KCAS**	*Knots Calibrated Air Speed*
equivalent	**KEAS**	*Knots Equivalent Air Speed*
True	**KTAS**	*Knots True Air Speed*

What Is Indicated Airspeed?

Principles of Simple Airspeed Indicators

Quite simply, Indicated Airspeed (IAS) is nothing more than the value of airspeed read off the airspeed instrument by the pilot. But the nature of IAS has some unique aspects to it:

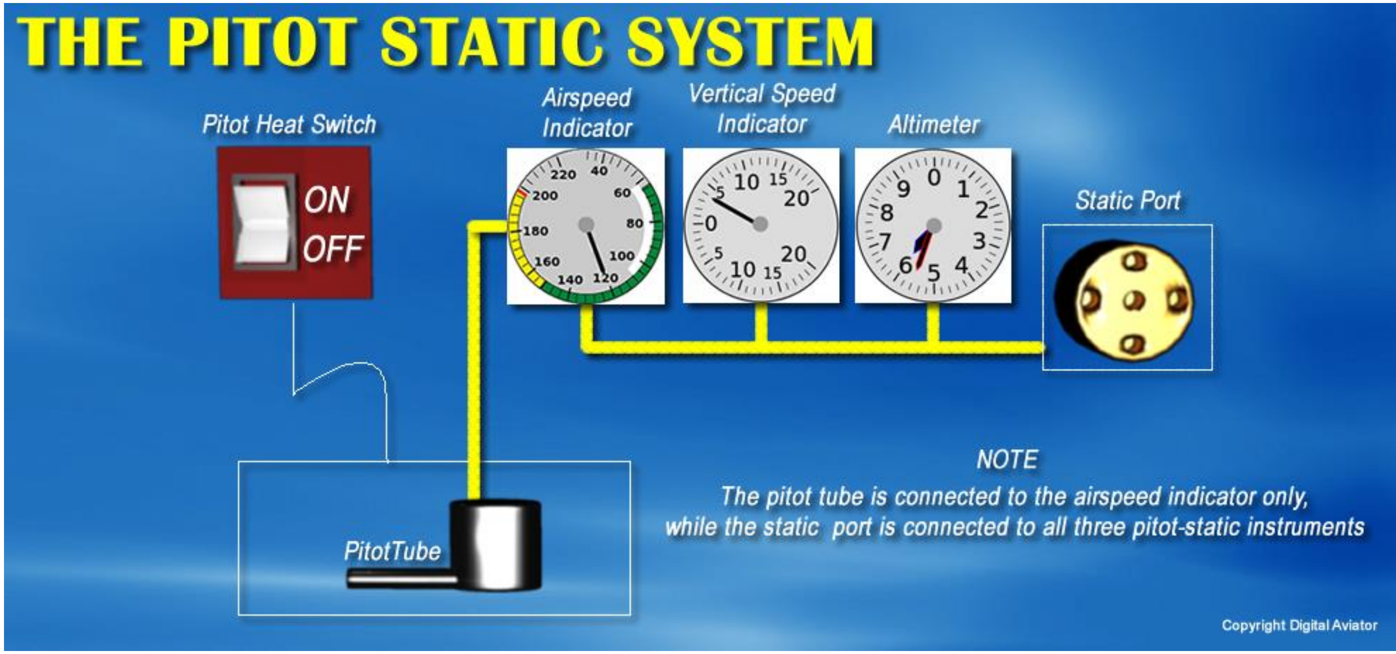
Indicator computes:

$$\bar{q} = P_T - P_s$$

Airspeed equation:

$$V_\infty = \sqrt{\frac{2*(P_T - P_s)}{\rho}}$$

SO WHAT ABOUT DENSITY???



Since density is impossible to measure directly, and difficult to compute accurately from imperfect measurements, all airspeed indicators are mechanized to use <u>standard day, sea level density</u> to display KIAS.

Determining Static Source Error Corrections

The Flight Test Trailing Pressure Cone

- ***There are errors in all physical measurements. So correcting the errors in measuring total and static pressures requires calibration of the pitot-static instruments and their computing element (air data computer):***

1) ***Total and static pressure measurements are subject to compressibility. Those are corrected under Equivalent Airspeed. (Next slides)***
2) ***Total pressure will experience errors as the airplane maneuvers, because the pitot tube will not necessarily be pointing directly into the relative wind at all times.***
3) ***Recall the airplane disturbs the natural flow of air, and causes pressure changes due to the flow around the aircraft. These cause static pressure measurement errors.***
4) ***Ideally, we would like to measure static pressure far ahead, or far behind, the air being disturbed by the airplane…***

Correcting Airspeed Measurement for Compressibility

We will learn about compressibility corrections to airspeed measurements after we take our midterm. Therefore, you will not be responsible for knowing how to solve the high speed equivalent and true airspeed problems for the midterm!

WARNING!!!

The Following Chart Contains Graphic Content Which Could Be Disturbing To Some (Non-Engineering) **Students.**

Viewer Discretion Is Advised!

Full-Blown Navier-Stokes Fluid Dynamics Equations

For 3-Dimensional, Unsteady Fluid Flows!

You shall learn & master these come Junior year (in ARO 301)

Coordinates: (x,y,z)

Velocity Components: (u,v,w)

Time : t

Density: ρ

Total Energy: Et

Pressure: p

Stress: τ

Heat Flux: q

Reynolds Number: Re

Prandtl Number: Pr

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X – Momentum:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$$

Y – Momentum:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$$

Z – Momentum

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$$

Energy:

$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z} - \frac{1}{Re_r Pr_r}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$
$$+ \frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y}(u\tau_{xy} + v\tau_{yy} + w\tau_{yz}) + \frac{\partial}{\partial z}(u\tau_{xz} + v\tau_{yz} + w\tau_{zz})\right]$$