Introduction to Aeronautics

ARO 101 Sections 03 & 04

Sep 30, 2015 thru Dec 9, 2015

Instructor: Raymond A. Hudson

Week #3 Lecture Material

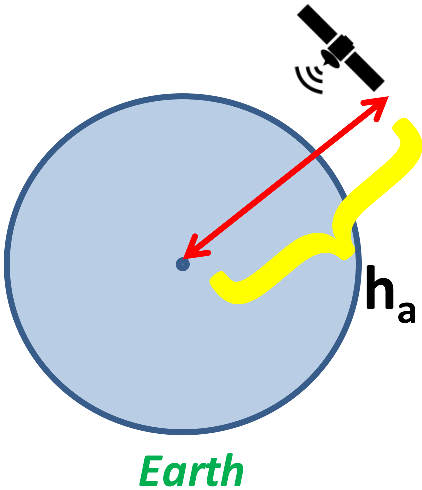
ARO 101 *Intro-Aero*

Topics For Week #3

- **<u>The Many Definitions of Altitude–</u> We can define and measure altitude in many different ways. We will describe all the various types of altitude measures we use in aerospace engineering.**
- **<u>Hydrostatics–</u> How does pressure vary with height in a non-moving fluid mass?** *The Hydrostatic Equation*
- **<u>Naming the Layers of Earth's Atmosphere-</u> There are FIVE layers in earth's atmosphere and each falls into 1 of the following 2 types:**
  - ✓ ***<u>Linear Gradient Layer:</u> Temperature changes linearly with height.***
  - ✓ ***<u>Isothermal Layer:</u> Temperature does not change in the layer.***
- **<u>Deriving Equations to Model the Atmosphere–</u> Use hydrostatic & ideal gas law equations to model how temperature, pressure, and density change with altitude in both linear gradient and constant temperature layers.** *International Standard Atmosphere*

Definitions Of Altitudes

(Each one is useful, but for different reasons)

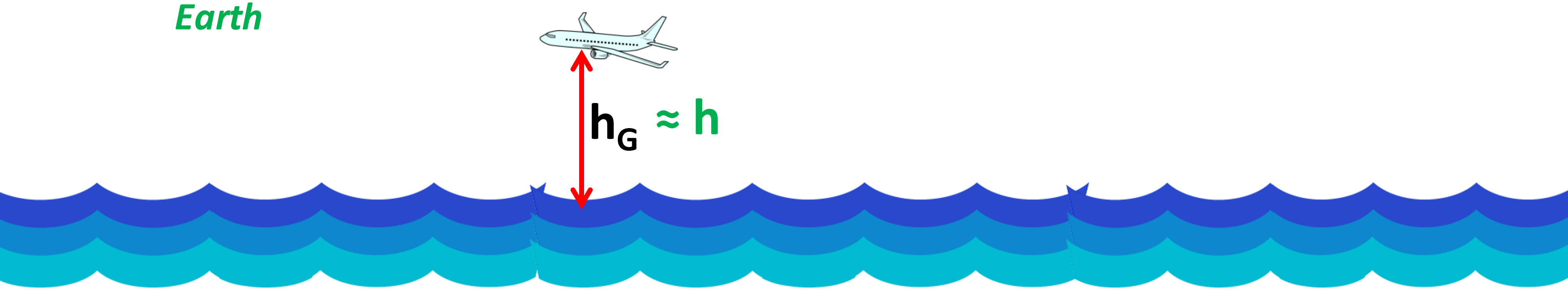


- ***ABSOLUTE ALTITUDE***

h_a – Distance from center of the earth to a body above it.

- ***GEOMETRIC ALTITUDE***

h_G – Actual, physical distance measured from mean sea level to the body above it. "Tape measure altitude."



- ***GEOPOTENTIAL ALTITUDE***

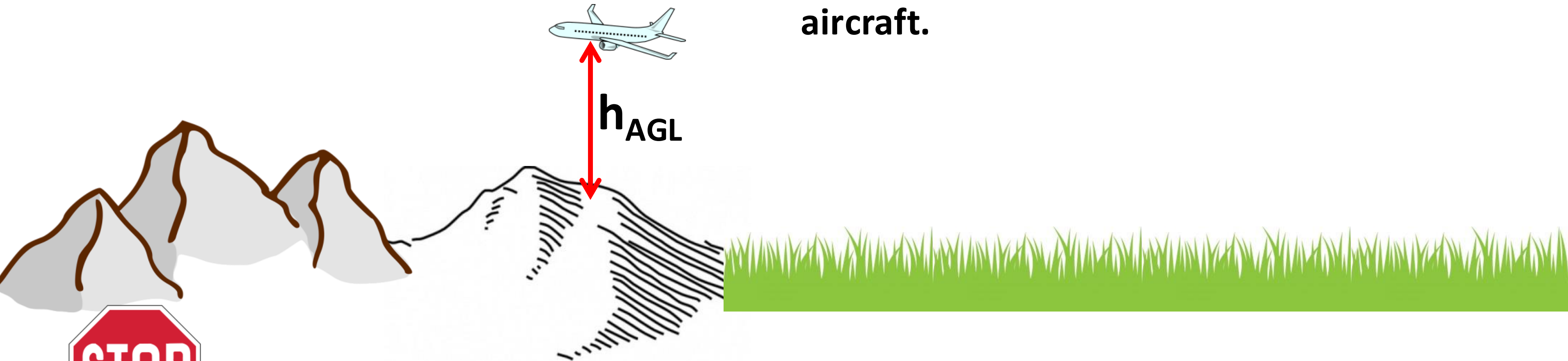
h – Approximation of geometric altitude which assumes constant gravity. "Fictitious altitude." Measurement of static pressure outside aircraft is used to calculate h.

Definitions Of Altitudes

(Continued)

- ***ABOVE GROUND LEVEL (AGL) ALTITUDE***

h_{AGL} – Distance from the local terrain to a body above it. Measured with a low power radar signal reflected off the terrain below aircraft.



Always Think: Appendices A & B in book

- ***PRESSURE ALTITUDE*** h_P – Pressure measured outside aircraft matches stated altitude for ISA Standard Day.
- ***TEMPERATURE ALTITUDE*** h_T – Temp measured outside aircraft matches stated altitude for ISA Standard Day.
- ***DENSITY ALTITUDE*** h_ρ – Density measured outside aircraft matches stated altitude for ISA Standard Day.

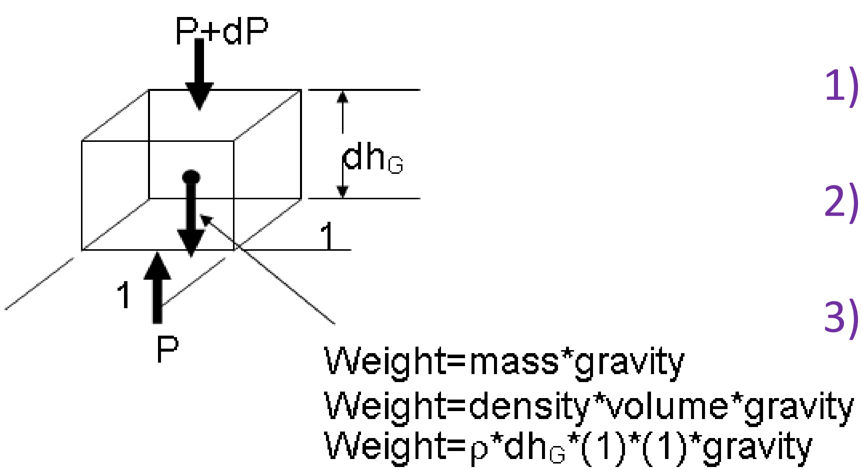
Hydrostatic Principles

(Using observation & analysis to quantify fluid pressure with depth)

- ***QUESTION:* Which of the following containers, which are all filled with water, has the highest pressure at the bottom?**

- ***ANSWER:* NONE! Pressure at the bottom of all of these containers is EQUAL!**
- ***BECAUSE:* Pressure is *ONLY* a function of *fluid density* and *depth*.**

***HOW* can we PROVE this is true?**

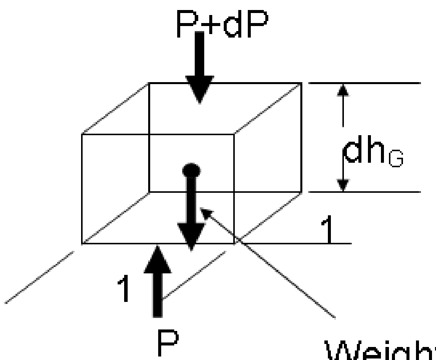


1) Assume a differential cube of the fluid, with unity length and width, and differential height “dh.”
2) Identify weight vector acting downward & write equation based on density, volume, and gravity.
3) Identify pressure forces acting on upper and lower surfaces of cube: P opposing (P+dP) over unit area.

AND THEN…

Hydrostatic Equation

(Continued)



• ***Since the fluid is in a static state (no motion), we recall that the summation of all forces must be zero.*** Σ ***Forces = 0***

Weight=mass*gravity
Weight=density*volume*gravity
Weight=ρ*dh_G*(1)*(1)*gravity

$$P*(1)*(1) - (P+dP)*(1)*(1) - \rho*dh*(1)*(1)*g_o = 0$$

Simplifying:

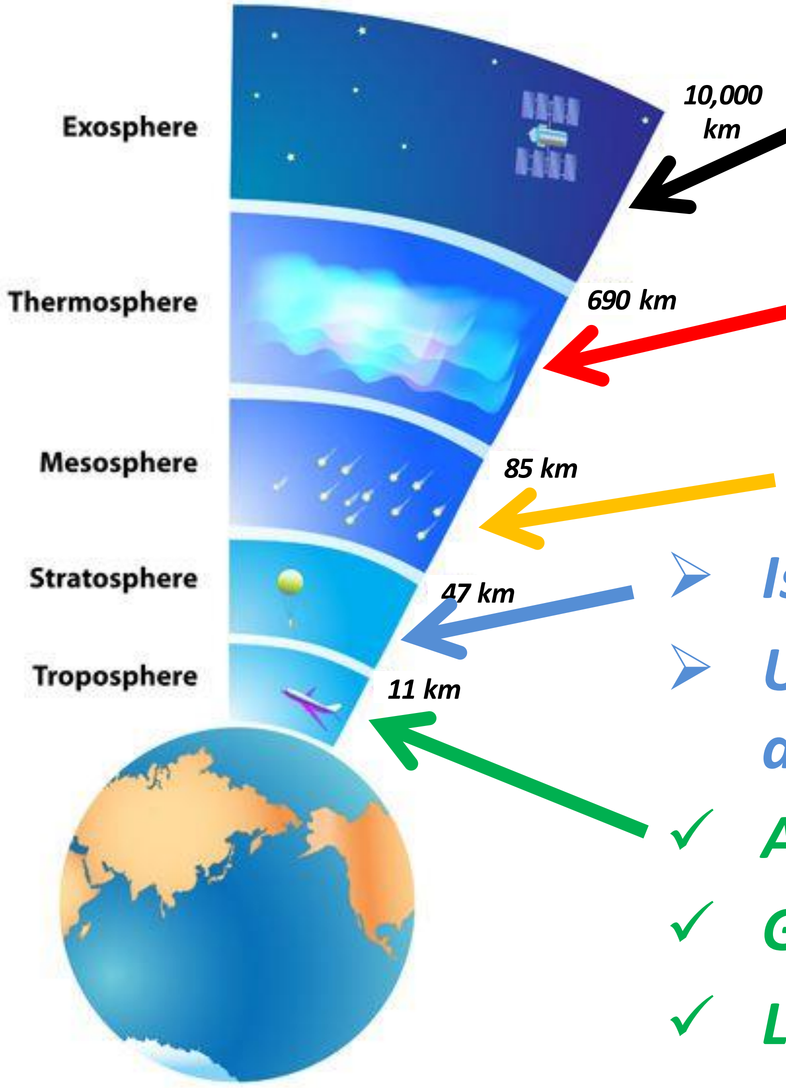
$$P - P - dP - \rho*g_o*dh = 0$$

The Hydrostatic Equation

$$\boxed{dP = -\rho*g_o*dh}$$

"As we move upward (+dh) in a fluid, the pressure decreases (-dP)"
"As we move downward (-dh) in a fluid, the pressure increases (+dP)"

Layers of Earth's Atmosphere



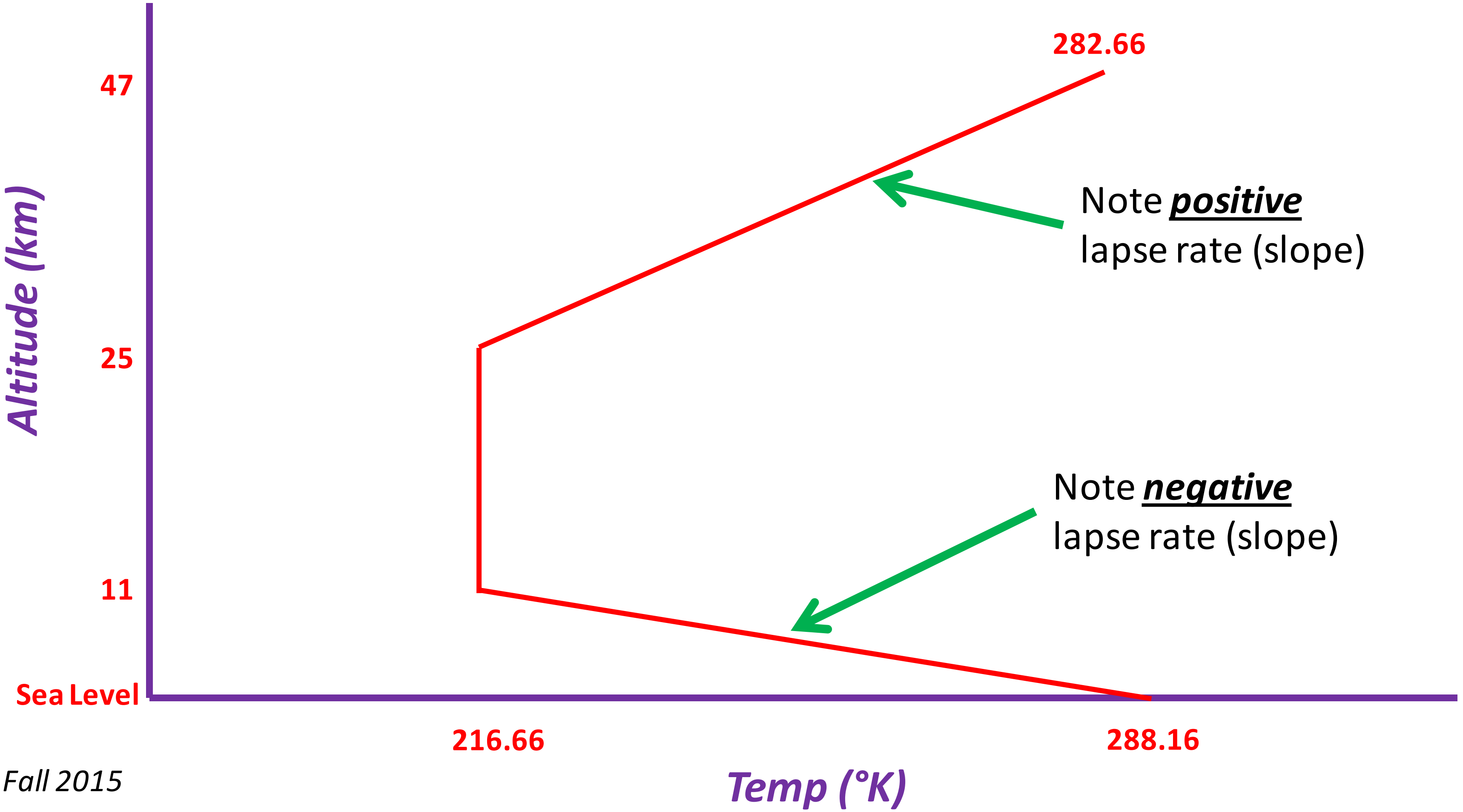
- Gradient layer, decreasing temps.
- Satellites. Virtually no atmosphere
- Isothermal layer, constant temps.
- Gases become ionized here.
- Gradient layer, increasing temps.
- Isothermal layer, constant temp.
- U2, Global Hawk & other reconnaissance and surveillance aircraft fly in this layer.
- ALL life and weather occurs in this layer.
- Gradient layer, decreasing temps.
- Large majority of aircraft fly in this layer.

ARO 101 *Intro-Aero*

ISA Temperature Profiles

(Standard Day Break Points)

Do not be confused by this plot! Altitude is the independent variable (X) and Temperature the dependent variable (Y). The plot is rotated to emulate altitude from mean sea level.



Pressure & Density In Isothermal Layers

- *Starting with the isothermal layer (constant temp = simpler) let us use our two equations:*

$$\frac{dP = -\rho * g_0 * dh}{P = \rho * R * T}$$

Hydrostatic Equation describes how pressure varies with height in the fluid.

Ideal Gas Law relates pressure to density and temperature at any point in the fluid.

Divide one equation by the other to form a combined relation.

$$\boxed{\frac{dP}{P}} = \boxed{\frac{-g_0}{(R*T)}} * dh$$

SIMPLIFIED FORM…and now we notice:

THIS TERM IS A CONSTANT VALUE

THIS TERM IS AN INTEGRAL FORM

SO LET'S GO AHEAD & INTEGRATE!

$$\int_{P_1}^{P_2} \frac{dP}{P} = \frac{-g_0}{(R*T)} * \int_{h_1}^{h_2} dh$$

Pressure & Density In STRATOSPHERE

• *We recognize (from our studies in MAT 115) that the integral form on the left side results in a natural log of the limits of the ratio of the integral limits. We also know the integral of "dh" is just the difference between the integral limits. Thus:*

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{-g_0}{(R*T)} * (h_2 - h_1)$$

• *As we solve the equation for the pressure ratio, let's see how we apply this general equation to the specific case of the STRATOSPHERE:*

P_2 = Pressure at whatever height within the stratosphere we desire.
P_1 = Pressure at base of stratosphere (tropopause) = P_{TP}
h_2 = Height within the stratosphere we desire to know the pressure.
h_1 = Height of the base of the stratosphere (tropopause) = h_{TP}

$$\frac{P}{P_{TP}} = e^{\left(\frac{-g_0}{(R*T)} * (h - h_{TP})\right)} = \boxed{\frac{\rho}{\rho_{TP}}}$$

AND

CAN YOU EXPLAIN WHY?

Pressure & Density In Gradient Layers

- ***To solve the equations for the gradient layers, we can still begin with the combined equation we developed for the isothermal layers. <u>But we must now realize that TEMPERATURE is no longer constant:</u>***

$$\frac{dP}{P} = \frac{-g_0}{(R*T)}*dh$$

We can write a linear equation to model the temperature in the gradient layer as:

$T_2 = T_1 + a_{LR}*(h_2 - h_1)$ **Where we define "a_{LR}" to be:** $a_{LR} = \frac{dT}{dh}$

Solve this equation for: $dh = \frac{dT}{a_{LR}}$

substitute

$$\int_{P_1}^{P_2} \frac{dP}{P} = \frac{-g_0}{(R*a_{LR})} * \int_{T_1}^{T_2} \frac{dT}{T}$$

Pressure & Density In TROPOSPHERE

- *Again, we recognize the integral forms of (dP/P) and (dT/T) to be natural log solutions, hence:*

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{-g_0}{(R*a_{LR})} * \ln\left(\frac{T_2}{T_1}\right)$$

- *As we solve the equation for the pressure ratio, let's see how we apply this general equation to the specific case of the TROPOSPHERE:*

P_2 = Pressure at whatever height within the troposphere we desire = P
P_1 = Pressure at base of troposphere (sea level) = P_{SL}
T_2 = Temp at point in troposphere where we desire to know the pressure = T
T_1 = Temp at the base of the troposphere (sea level) = T_{SL}

$$\frac{P}{P_{SL}} = \left(\frac{T}{T_{SL}}\right)^{\frac{-g_0}{(R*a_{LR})}} \quad \textbf{AND} \quad \frac{\rho}{\rho_{SL}} = \left(\frac{T}{T_{SL}}\right)^{-\left(\frac{g_0}{(R*a_{LR})} + 1\right)}$$

ARO 101
Intro-Aero

Fluid Compressibility & Mach Number

(Mach >= 0.3, Air Is Compressible!)

- ***Next week, we will begin learning the basics of fluid dynamics. Understanding when we can consider air incompressible, and when we must treat is as compressible is important!***

Air MUST be treated as compressible when flow Mach Number $\geq$ 0.3

- ***So the next question becomes: WHAT IS MACH NUMBER?***
- ***MACH NUMBER – The ratio of a fluid's flow velocity to the local speed of sound in the fluid medium.***

$$\textit{Mach (M)} = \frac{V_\infty}{a_{SOS}} = \frac{\text{Fluid Speed (i.e. Airspeed)}}{\text{Speed of Sound in Fluid}}$$

$$a_{SOS} = \sqrt{\gamma * R * T}$$

Specific Gas Constant

Fluid Temperature

Ratio of Specific Heats (air = 1.4)